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World-volume fields, $\mathrm{SL}(2;\mathbb{Z})$ and duality: the type II B 3-brane

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Abstract: We propose a method for constructing super-brane actions where every background tensor potential corresponds to a world-volume field strength. The procedure provides a natural coupling to the background and automatically displays the $\mathrm{SL}(2;\mathbb{Z})$ symmetry of the IIB string theory. The Dirichlet 3-brane is used as a test ground for these ideas. A polynomial action consistent with non-linear self-duality is presented. Invariance of the action under κ-symmetry is demonstrated for arbitrary on-shell type IIB supergravity backgrounds and is shown to require self-duality.

Keywords: Branes in String Theory, Space-Time Symmetries.

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Contents

1 Introduction

Extended supersymmetric objects arise as soliton configurations in the low-energy effective field theories of string theory and M-theory. Most profoundly, the solitons of type II string theory are realised as D-branes [1], i.e., objects on which elementary strings may end [2]. Many aspects of the dynamics of these objects, which is the main subject of this paper, are by now fairly well understood. For instance, it was shown early on that the fact that elementary strings may end on a D-brane necessitates the presence of a self-interacting vector potential on the D-brane world-volume with its dynamics governed by the Dirac–Born–Infeld (DBI) action [3]. More recently, the world-volume dynamics of supersymmetric D-branes has been completely formulated [4, 5, 6, 7]. An important ingredient in the latter constructaion is κ-symmetry, a fermionic gauge symmetry reflecting the fact that these configurations are BPS-saturated and break half the space-time supersymmetries.

A serious drawback of the picture of refs. [4, 5, 6, 7] for the case of type IIB branes is that it breaks the SL(2;$\mathbb{Z}$) S-duality symmetry [8] of the type IIB theory. This shortcoming has more or less been taken for granted, since SL(2;$\mathbb{Z}$) is a non-perturbative symmetry and a choice of elementary excitations is required; indeed, the non-covariance is inherent in the very notion of D-branes as objects on which elementary open strings may end. Nevertheless, recent developments [9, 10, 11] have shown that by treating simultaneously the entire SL(2;$\mathbb{Z}$) multiplet of type IIB superstrings as "elementary", a formulation that makes the SL(2;$\mathbb{Z}$) symmetry manifest can be achieved. While such a description should not be expected to be possible in terms of local field theory for a theory possessing a strong-weak coupling symmetry involving the interchange of electric and magnetic charges (such as, e.g., N=4 super Yang–Mills theory), the string theory symmetry does not involve any such charge interchange, and there seems to be no immediate obstruction to an SL(2;$\mathbb{Z}$)-covariant *perturbative* formulation.

The other branes of the type IIB theory also come in SL(2;$\mathbb{Z}$) multiplets, and one would like to find a description that respects this covariance, generalising the superstring action of ref. [11]. The present paper makes a concrete proposal for how this may be accomplished. Our main proposal is that any tensor potential in the background supergravity theory should correspond to a world-volume field strength containing the background potential schematically as "$F = dA - C$", thus generalising the mechanism required to achieve gauge invariance in the traditional formulation. If the background theory (type IIB supergravity) is formulated with manifest SL(2;$\mathbb{R}$) symmetry, the same will then be true for the classical brane action. Quantum effects on the superstring world-sheet break SL(2;$\mathbb{R}$) to SL(2;$\mathbb{Z}$) [8, 12, 11].

The formulation is not limited to type IIB, but should have its most important application there. In addition to displaying more symmetry, it should provide clearer information about branes ending on other branes, in which case the boundary should couple directly to a tensor potential on the brane. It may also simplify the formulation in other cases (type IIA and 11 dimensions), since it in general seems to yield polynomial actions.

In the present paper, our proposal is tested on the 3-brane of type IIB string theory, which is a singlet under SL(2;$\mathbb{Z}$). This fact has previously been shown to correspond to certain duality properties of the DBI action [13], and those results are now recast in a completely covariant and SL(2;$\mathbb{Z}$)-symmetric form, including the coupling to the background. The type IIB 5-brane case, which will be even more interesting since the charges come in an SL(2;$\mathbb{Z}$) doublet, will follow a similar pattern and will be worked out in a separate paper.

We should mention that the action we write down does not imply the non-linear self-duality constraint that is imposed on the world-volume 2-form field strength. There are methods for incorporating self-duality so that it follows from an action principle [14], but none of these seem to clarify any of the points we want to make. Indeed, consistency of self-duality with the Bianchi identities and equations of motion turns out to be a powerful instrument by itself. If self-duality can be handled in functional integration, as described by Witten for the (linearised) 11-dimensional 5-brane [15], a formulation like the present one is equally useful.

The disposition of the paper is as follows. In section 2 we recapitulate some essentials of type IIB supergravity, and work out the exact structure of the world-volume fields. Section 3 deals with the non-linear self-duality relevant for the 3-brane, and casts it into a covariant form. In section 4 the 3-brane action is given, and in section 5 κ-symmetry is used to derive the same form of the action. Finally, section 6 contains a discussion of our results together with some conjectures on the further application of our method. Some notes on notation are found in an appendix.

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2 World-volume and background fields

We advocate the proposal that coupling of branes to background tensor potentials of any rank should be achieved solely through the formation of world-volume field strengths of the type "$F = dA - C$", C being (the pullback of) a background potential with field strength "$H = dC$" and A a world-volume potential. The quotation marks are meant to indicate that these relations are to be seen as schematic; the actual background field theories, supergravities in 10 or 11 dimensions, contain "modified" fields strengths, a fact which will imply corresponding deviations from the schematic expressions above also on the world-volume. We will concretise all this in a little while for the type IIB theory; we expect the type IIA and 11-dimensional cases to behave similarly.

Before investigating the structure of the world-volume fields, some words need to be said about the background. In the present paper, we will focus on the case of the 3-brane in type IIB string theory and the relevant background fields will be those of type IIB supergravity [16, 17].

The type IIB branes are embedded in chiral complex (10|32) superspace with coordinates $Z^M = (X^m, \theta^\mu, \bar{\theta}^{\bar{\mu}})$. Local Lorentz indices are $A = (a, \alpha, \bar{\alpha})$. To avoid confusion, note that although the indices α and $\bar{\alpha}$ denote components of complex spinors, we may still use real (Majorana) γ-matrices, which makes them behave identically under local Lorentz transformations. In some cases, such as the supergravity constraints of section 5, objects with seemingly different index structures may thus be identified.

The bosonic sector of type IIB supergravity consists, apart from gravity, of scalar fields in the coset $\mathrm{SL}(2;\mathbb{R})/\mathrm{SO}(2)$, an $\mathrm{SL}(2;\mathbb{R})$ doublet of 2-form potentials C_r and an $\mathrm{SL}(2;\mathbb{R})$ singlet 4-form potential C with self-dual field strength. The formulation of this theory is well known, and we will only briefly state some relevant points in order to establish our notation.

The scalars are represented as a complex SL(2;$\mathbb{R}$) doublet $\mathscr{U}^r$ constrained by $\frac{i}{2}\varepsilon_{rs}\mathscr{U}^r\bar{\mathscr{U}}^s = 1$. The coset is obtained by gauging the U(1) acting as $\mathscr{U}^r \to e^{i\vartheta}\mathscr{U}^r$. We normalise the U(1) charge e to 1 for the scalars $\mathscr{U}^r$. The physical dilaton and axion fields ϕ and χ are identified as the projective invariant $\mathscr{U}^2/\mathscr{U}^1 = \chi + ie^{-\phi}$, but we will stick to the $\mathrm{SL}(2;\mathbb{R})$-covariant notation. If the above action of $\mathrm{SL}(2;\mathbb{R})$ on the scalars is considered the left action, there is also a right action, of which the U(1) is a subgroup. The left-invariant Maurer–Cartan forms and equations are

$$\begin{aligned} Q &= \frac{1}{2}\varepsilon_{rs}d\mathscr{U}^r\bar{\mathscr{U}}^s\,, \quad dQ - iP\wedge\bar{P} = 0\,, \\ P &= \frac{1}{2}\varepsilon_{rs}d\mathscr{U}^r\mathscr{U}^s\,, \quad dP - 2iP\wedge Q \equiv DP = 0\,, \end{aligned} \tag{2.1}$$

where the covariant derivative D (acting from the right) includes the real U(1) connection Q as $D = d - ieQ$, e denoting the U(1) charge.

The 3-form field strengths are formed as $H_r = dC_r$. The scalars act as a bridge between objects that are $\mathrm{SL}(2;\mathbb{R})$ doublets and U(1) singlets, such as H_r, and the corresponding objects which are $\mathrm{SL}(2;\mathbb{R})$ singlets but carry U(1) charge 1, such as $\mathscr{H} \equiv H_r\mathscr{U}^r$. In this function they are necessary for the existence of SL$(2;\mathbb{R})$-invariant kinetic terms like $\mathscr{H}\cdot\bar{\mathscr{H}}$ (for the $\theta=0$ components). We are using roman versus script letters to make the same distinction also for other fields. The complex field strength satisfies the Bianchi identity

$$dH_r\mathscr{U}^r \equiv D\mathscr{H} + i\bar{\mathscr{H}}\wedge P = 0\,. \tag{2.2}$$

The 5-form field strength and Bianchi identity are

$$H = dC + \mathrm{Im}\,(C\wedge\bar{\mathscr{H}})\,, \qquad dH - i\mathscr{H}\wedge\bar{\mathscr{H}} = 0\,. \tag{2.3}$$

where the "modifying" terms come from consistency of self-duality with the presence of the supergravity "Chern–Simons" term $\propto i\int\mathscr{H}\wedge\bar{\mathscr{H}}\wedge C$. A corresponding statement of course applies for the 7-form, whose $\theta=0$ component is dual to the 3-form. Analogous consistency conditions imposed on world-volume fields will be central for the continued discussion.

With these preliminaries at hand, we are ready to treat world-volume fields defined by modifications of the typical relation "$F=dA-C$". The guiding principle is gauge invariance, both with respect to background and world-volume gauge transformations. Up to trivial rescalings, one is uniquely led to the 2-form and 4-form field strengths

$$\mathscr{F} = dA_r\mathscr{U}^r - \mathbb{C}\,, \qquad F = dA - C + \mathrm{Im}\,(\mathscr{A}\wedge\bar{\mathscr{H}})\,, \tag{2.4}$$

with the Bianchi identities

$$D\mathscr{F} + i\bar{\mathscr{F}}\wedge P = -\mathscr{H}\,, \qquad dF = -H - \mathrm{Im}\,(\mathscr{F}\wedge\bar{\mathscr{H}})\,. \tag{2.5}$$

It is straightforward to verify that all field strengths are invariant under the gauge transformations

$$\begin{aligned} \delta C_r &= dL_r\,,\\ \delta C &= \mathrm{Im}\,(\mathscr{L}\wedge\bar{\mathscr{H}}) + dL\,,\\ \delta A_r &= dl_r + L_r\,,\\ \delta A &= \mathrm{Im}\,(\ell\bar{\mathscr{H}}) + dl + L\,, \end{aligned} \tag{2.6}$$

where $\mathscr{L}\equiv L_r\mathscr{U}^r$ and L are background 1- and 3-form gauge parameters, while the world-volume parameters $\ell\equiv l_r\mathscr{U}^r$ and l are 0- and 2-forms. These definitions are independent of the dimension of the world-volume. For the case of a 3-brane these fields are sufficient; for higher-dimensional p-branes the procedure will and can be continued up to the field strength of rank $p+2$.

The number of world-volume fields and their transformation properties are thus completely determined by the structure of the background supergravity theory, in this case type IIB. We know, on the other hand, e.g. by comparison to the field content of D-branes, that this in general gives too many bosonic degrees of freedom. Apart from transverse oscillations, type II branes contain one vector potential. The 3-brane, following our proposal, carries an SL$(2;\mathbb{R})$ doublet of vector potentials and a 3-form without local degrees of freedom, while a 5-brane has the vector doublet, the 3-form singlet and a doublet of 5-form potentials without local degrees of freedom. Thus, there must be a mechanism by which the number of physical fields is reduced to the correct one. This mechanism is a kind of duality relation, which will be investigated for the 3-brane in the next section. We would like to remark that even if the proposed set of fields containing background couplings may not be seen as necessary, manifest SL(2) symmetry requires at least the doublet of vector potentials, which already calls for some relation between the field strengths. The exception is the case $p=1$ treated in refs. [10, 11], where the vector potentials do not carry any local degrees of freedom.

3 Self-duality

The duality properties of the DBI action for the 3-brane are well understood; the 3-brane is invariant under a simultaneous duality transformation of the world-volume vector field and a transformation of the background fields under the non-trivial element in the $\mathbb{Z}_2$ subgroup of $\mathrm{SL}(2;\mathbb{Z})$ interchanging NS-NS and RR charges [13]. The scalar fields have also been included in this transformation [18], but not in a manifestly $\mathrm{SL}(2;\mathbb{Z})$-covariant way. Neither have the duality properties been examined in the supersymmetric case, though it is obvious that they must hold. We will cast the known duality transformations into a covariant form, to be used later for a manifestly SL(2)-covariant formulation of the supersymmetric 3-brane.

The 3-brane DBI action

$$S_{\mathrm{DBI}} = -\int d^4\xi\,\sqrt{-\det(g+F)}\ , \tag{3.1}$$

contains one real 2-form field strength F. The dual field strength G, defined by

$$*G = \pm 2\frac{\delta S_{\mathrm{DBI}}}{\delta F}\ ,$$

is related to F by the non-linear duality transformation

$$G = \pm N^{-1}(F)\left(*F + \frac{1}{2}(F\cdot *F)F\right)\ , \tag{3.2}$$

with inverse

$$F = \mp N^{-1}(G)\left(*G + \frac{1}{2}(G\cdot *G)G\right)\ , \tag{3.3}$$

where $N(F) = (1 + F\cdot F - \frac{1}{4}(F\cdot *F)^2)^{1/2}$ (note that $\sqrt{-g}N(F) = -\mathscr{L}_{\mathrm{DBI}}$).

We would like to associate these real fields with the real components of the field strength $\mathscr{F}$ of the previous section. The first critical step to be taken is to check whether (3.2) respects the U(1) symmetry. The duality transformation is not manifestly U(1) symmetric, but it is quite straightforward to show that it holds unchanged also for the real components of $e^{i\vartheta}(F+iG)$. Disregarding the physical reasons for identifying $\mathscr{F}=F+iG$, this is quite astonishing, since (3.2) contains an infinite number of terms with different

U(1) charges. The next task is to find the manifestly U(1)-symmetric formulation of the duality. One piece of information is found in the observation that $G\cdot{*G} = -F\cdot{*F}$, a relation which takes the U(1)-covariant form

$$\mathscr{F}\cdot{*\bar{\mathscr{F}}} = 0\,. \tag{3.4}$$

We therefore make an ansatz for self-duality consistent with this identity:

$$i{*\mathscr{F}} = \alpha\mathscr{F} + \beta\left((\mathscr{F}\cdot\mathscr{F})\bar{\mathscr{F}} + (\mathscr{F}\cdot{*\mathscr{F}}){*\bar{\mathscr{F}}}\right)\,, \tag{3.5}$$

where α and β are uncharged scalar functions of $\mathscr{F}$. This ansatz must be restricted in order to be consistent (eq. (3.5) encodes both the duality transformation (3.2) and its inverse (3.3)). In order to shorten the expressions, we introduce the quadratic invariants $r\equiv\mathscr{F}\cdot\bar{\mathscr{F}}$, $p\equiv\mathscr{F}\cdot\mathscr{F}$, $q\equiv\mathscr{F}\cdot{*\mathscr{F}}$ at U(1) charges 0, 2 and 2, respectively. Consistency of the non-linear self-duality demands that the relations

$$\begin{aligned} q &= -i(\alpha+\beta r)p \quad \left(= -\tfrac{i}{\alpha-\beta r}p\right)\,, \\ r &= i\beta(p\bar{q}-\bar{p}q) \quad \left(= -\tfrac{\beta}{\alpha}(p\bar{p}+q\bar{q})\right)\,, \\ \alpha^2 &- \beta^2 r^2 = 1 \end{aligned} \tag{3.6}$$

be satisfied. This leaves only one independent complex invariant, matching in number the two scalars $F{\cdot}F$ and $F{\cdot}{*F}$. The remaining freedom resides in the choice of the function β, which will be determined from the actual duality (3.2). Since we have U(1) covariance, we may choose any equation following from (3.2) but not implied by (3.4) that we wish. Even if it consists of terms with different charge e, we know that it will be fulfilled separately at each value of e. We chose to use the symmetric equation $N(F)N(G) = 1 + \frac{1}{16}(\mathrm{Re}\,q)^2$. We also checked explicitly that any of the charge components of this equation implies

$$\alpha = \pm\frac{1+\frac{r}{8}}{\sqrt{1+\frac{r}{4}}}\,, \qquad \beta = \mp\frac{1}{8\sqrt{1+\frac{r}{4}}}\,. \tag{3.7}$$

The duality may be simplified further; one useful observation, that we will employ in the proof of κ-symmetry as well, is that when the self-duality constraint is satisfied, each of the matrices $\mathscr{F}_i{}^j$, $\bar{\mathscr{F}}_i{}^j$, ${*\mathscr{F}_i{}^j}$ and ${*\bar{\mathscr{F}}_i{}^j}$ is expressible as a sum with scalar coefficients of odd powers of only one real matrix $F_i{}^j$, and, consequently, they all commute. Other relations that follow are (matrix multiplication) $\mathscr{F}{*\mathscr{F}} = \frac{1}{4}\mathrm{tr}(\mathscr{F}{*\mathscr{F}})1\!\!1$ and $\mathscr{F}{*\bar{\mathscr{F}}} = -{*\mathscr{F}}\bar{\mathscr{F}}$. The final form of the self-duality relation is

$${*\mathscr{F}} = \frac{\mp i}{\sqrt{1+\frac{1}{4}\mathscr{F}\cdot\bar{\mathscr{F}}}}\left(\mathscr{F} - \frac{1}{4}(\mathscr{F}\cdot{*\mathscr{F}}){*\bar{\mathscr{F}}}\right)\,. \tag{3.8}$$

In the following section we will formulate an action that is consistent with this relation.

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4 The 3-brane action

There is a number of qualifications we require of the action, the first one being, of course, that it be equivalent to the DBI action. This may only be achieved if the action is compatible with the self-duality relation (3.8), which is what guides us in the first place. We also want manifest SL(2)-covariance, which is achieved by using only the fields of section 2. Finally, we need κ-symmetry to get the correct number of fermions on the world-volume.

In the spirit of earlier formulations of strings and membranes with p-form world-volume potentials [19, 10, 11], we make an ansatz for the action as

$$S = \int d^4\xi\, \lambda\sqrt{-g}\left\{1 + \Phi(\mathscr{F}, \bar{\mathscr{F}}) + cF\cdot F\right\}\,, \tag{4.1}$$

where λ is a Lagrange multiplier field and c a numerical constant. In order to judge the consistency of self-duality, we need to compare Bianchi identities and equations of motion for the field strength $\mathscr{F}$. To this end, define $\mathscr{K} = \frac{\partial\Phi}{\partial\mathscr{F}}$. Noticing that the last term in the action (4.1) may be recast in the alternative forms $-c\int d^4\xi\lambda\sqrt{-g}(*F)^2 = -c\int\lambda F\wedge *F$, the equation of motion for λ is readily found to be

$$*F = \pm\frac{1}{\sqrt{c}}\sqrt{1+\Phi}\,, \tag{4.2}$$

while the one for A reads $d(\lambda *F) = 0$. Furthermore, taking into account the fact that F contains $\mathscr{A}$, the equations of motion for A_r may be written

$$D(\lambda *\mathscr{K}) - i\lambda *\bar{\mathscr{K}}\wedge P = ic\lambda(*F)\mathscr{H}\,, \tag{4.3}$$

which should be compared to the Bianchi identity $D\mathscr{F} + i\bar{\mathscr{F}}\wedge P = -\mathscr{H}$. The only duality relation consistent with the equations of motion following from the action (4.1), in the sense that the background field $\mathscr{H}$ cancels, is

$$*\mathscr{K} = -ic\mathscr{F}(*F)\,. \tag{4.4}$$

Note that the sign of the term containing the Maurer–Cartan form P changes under multiplication of the field with i. This is the only check we need to perform in order to see that duality works for arbitrary background dilaton and axion fields. Matching eq. (4.4) with the results of the previous section, we arrive at $c = \frac{1}{4}$ (the actual number being, of course, a consequence of the choice of normalisation in (2.3)) and

$$S = \int d^4\xi\, \lambda\sqrt{-g}\Big\{1 + \frac{1}{2}\mathscr{F}\cdot\bar{\mathscr{F}} - \frac{1}{16}(\mathscr{F}\cdot *\mathscr{F})(\bar{\mathscr{F}}\cdot *\bar{\mathscr{F}}) + \frac{1}{4}F\cdot F\Big\}\,. \tag{4.5}$$

The duality now takes the polynomial form

$$\frac{i}{2}(*F)*\mathscr{F} = \mathscr{F} - \frac{1}{4}(\mathscr{F}\cdot *\mathscr{F})*\bar{\mathscr{F}}\,. \tag{4.6}$$

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Note that once the duality is satisfied, the quartic term in (4.5) reduces to $-\frac{1}{4}\mathscr{F}\cdot\bar{\mathscr{F}}$, so that the square root in (4.2) becomes identical to the one in (3.8) (this relation should of course not be inserted into the action). It is, in fact, not necessary to determine the unknown ingredients in (4.1) by comparison with earlier results. One may as well go directly to the following section and get the form (4.5) directly from the requirement of κ-symmetry.

Let us end the present section with a comment on the fact that there is no Wess–Zumino term in the action. Considering the actual form of F and its equation of motion, a partial integration is sufficient to see that the F^2 term plays, in fact, the same rôle as the Wess–Zumino term does in the "old" D3-brane action (although there is a difference in that the present term includes both 2-form field strengths). This statement is true in general for the $(p+1)$-form field strengths. The sign of the duality (4.6) depends on the sign of $*F$ in eq. (4.2). Therefore the present action describes two sectors of solutions, where the 2-form is self-dual or anti-self-dual, i.e., 3-branes and anti-3-branes. In previous descriptions the two sectors differ by the sign of the Wess-Zumino term, or, equivalently, of the "RR" charge (the notion is really an artefact of the $SL(2;\mathbb{Z})$-non-covariant choice of elementary string excitations; for the 3-brane this charge is $SL(2;\mathbb{Z})$-invariant).

5 κ-symmetry

κ-symmetry is the invariance under the transformations induced by the local fermionic translations $\delta_\kappa Z^M = \kappa^\alpha E_\alpha{}^M + \bar{\kappa}^{\bar{\alpha}} E_{\bar{\alpha}}{}^M$. In particular, for a background form pulled back to the world-volume the induced transformation reads $\delta_\kappa\Omega = \mathscr{L}_\kappa\Omega = (i_\kappa d + d i_\kappa)\Omega$. Transformations of world-volume fields must be specified separately, with the purpose of making the transformations gauge-covariant. To this end, the potentials of section 2 are taken to transform as

$$\begin{aligned} \delta_\kappa A_r &= i_\kappa C_r\,, \\ \delta_\kappa A &= i_\kappa C - \mathrm{Im}\,(\mathscr{A}\wedge i_\kappa\bar{\mathscr{H}})\,, \end{aligned} \tag{5.1}$$

which results in the following transformations of the world-volume field strengths:

$$\begin{aligned} \delta_\kappa\mathscr{F} &= -i\bar{\mathscr{F}}\wedge i_\kappa P - i_\kappa\mathscr{H}\,, \\ \delta_\kappa F &= -i_\kappa H - \mathrm{Im}\,(\mathscr{F}\wedge i_\kappa\bar{\mathscr{H}})\,. \end{aligned} \tag{5.2}$$

One also has to consider the transformation of the induced metric,

$$\delta_\kappa g_{ij} = 2E_{(i}{}^a E_{j)}{}^B \kappa^\alpha\, T_{B\alpha}{}^b \eta_{ab} + \text{c.c.} \tag{5.3}$$

The gauge parameter κ is subject to some restriction that makes it gauge away only half the fermions; an essential part of establishing κ-symmetry amounts to finding the half-rank matrix annihilating κ.

When the above transformation rules are applied in the variation of an action, one needs explicit expressions for the background field components involved. The latter are only those carrying at least one spinor index, and as such they are subject to constraints.

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A convenient set of constraints, which is consistent with the Bianchi identities of section 2 as well as with those for the torsion and implies the equations of motion for the background fields, is at dimension 0:

$$
\begin{aligned}
T_{\alpha\bar{\beta}}{}^{a} &= (\gamma^{a})_{\alpha\beta}\,, \\
\mathscr{H}_{a\alpha\beta} &= 2(\gamma_{a})_{\alpha\beta}\,, \\
H_{abc\alpha\bar{\beta}} &= 2i(\gamma_{abc})_{\alpha\beta}\,,
\end{aligned}
\tag{5.4}
$$

and at dimension 1/2:

$$
\begin{aligned}
&H_{abcd\alpha} = 0\,, \\
&\mathscr{H}_{ab\bar{\alpha}} = -i(\gamma_{ab}P)_{\alpha}\,, \\
&T_{\bar{\alpha}\bar{\beta}}{}^{\gamma} = i\delta_{(\alpha}{}^{\gamma}P_{\beta)} - \tfrac{i}{2}(\gamma_{a})_{\alpha\beta}(\gamma^{a}P)^{\gamma}\,, \\
&P_{\bar{\alpha}} = 0\,, \qquad Q_{\alpha} = 0\,;
\end{aligned}
\tag{5.5}
$$

the constraints at dimension 1 and above do not enter in the κ-variation. Components of the appropriate dimensions not appearing in (5.4) or (5.5) vanish. Also, we do not separately give components that follow by complex conjugation.

At this point, it is very convenient to take advantage of the fact that we have a duality relation; since the variation of the action, irrespective of the exact form of the latter, will contain the fields $\mathscr{K}$ of section 4, we may use self-duality in the form (4.4) to obtain the exact variation whatever the function Φ may be. The variation of the Lagrange multiplier is as usual unimportant, so we content ourselves with varying the constraint $\Psi \equiv 1 + \Phi + \frac{1}{4}F\cdot F \approx 0$. The result is at dimension 0 and 1/2, respectively,

$$
\begin{aligned}
(\delta_{\kappa}\Psi)_{(0)} &= (*F)\bar{E}_{i}\left\{\left[\tfrac{1}{2}(*F)\gamma^{i} - \tfrac{\mathrm{i}}{6\sqrt{-g}}\varepsilon^{ijkl}\gamma_{jkl} + \tfrac{i}{2}(*\mathscr{F}\bar{\mathscr{F}})^{ij}\gamma_{j}\right]\kappa + i*\mathscr{F}^{ij}\gamma_{j}\bar{\kappa}\right\} + \text{c.c.}\,, \\
(\delta_{\kappa}\Psi)_{(1/2)} &= \tfrac{1}{8}(*F) * \mathscr{F}^{ij}\bar{P}_{\alpha}\left[\mathscr{F}_{ij}\bar{\kappa} - 2(\gamma_{ij}\kappa)\right]^{\alpha} + \text{c.c.}
\end{aligned}
\tag{5.6}
$$

We must then find a matrix M of half maximal rank such that this variation vanishes when "$\kappa = M\zeta$" (real matrix acting on real components) for an unconstrained spinor ζ. Indeed, if we let

$$
\kappa = \left(\frac{\mathrm{i}}{24\sqrt{-g}}\varepsilon^{ijkl}\gamma_{ijkl} + \frac{1}{2}*F\right)\zeta - \frac{i}{4}*\mathscr{F}^{ij}\gamma_{ij}\bar{\zeta}\,,
\tag{5.7}
$$

we find that both parts of (5.6) vanish, provided that $1 + \frac{1}{4}\mathscr{F}\cdot\bar{\mathscr{F}} + \frac{1}{4}F\cdot F = 0$ and the duality relation (4.6) holds. It is not difficult to see that eq. (5.7) can be written as a projection $\kappa = P_{+}\kappa$, where the projectors $P_{\pm}$ are given by

$$
(*F)P_{\pm}\psi = *\left(\frac{1}{2}F\psi \mp \frac{i}{2}\mathscr{F}\wedge\gamma_{(2)}\bar{\psi} \pm i\gamma_{(4)}\psi\right)\,.
\tag{5.8}
$$

The invariance under κ-symmetry thus demands the same action and the same self-duality relation as were derived earlier using the DBI action as input.

6 Discussion

We have demonstrated how the type IIB string theory super-3-brane, by the introduction of a full set of world-volume fields corresponding to the background ones, can be given a formulation where the SL(2;$\mathbb{Z}$) symmetry becomes manifest. The construction involved a certain non-linear self-duality relation needed to reduce the number of physical degrees of freedom to the correct one, and also demanded by supersymmetry. We take this as a strong indication that the idea of introducing the higher world-volume tensors is indeed a good one. The final test will of course be the multiplet of 5-branes, but it is quite clear from the present investigation how it will work. In this case we start from an ansatz of the form

$$S=\int d^6\xi\,\lambda\sqrt{-g}\,\big\{1+\Phi(\mathscr{F},\bar{\mathscr{F}},F)+c\,\mathscr{G}\!\cdot\!\bar{\mathscr{G}}\big\}\ ,$$

where $\mathscr{G}=dA_{(5)r}\mathscr{U}^r-\mathbb{C}_{(6)}+\frac{1}{3}A\wedge\mathscr{H}-\frac{1}{6}\mathscr{A}\wedge H$ is a non-dynamical complex 6-form. Again, there will be implicit restrictions analogous to (4.4) on possible duality relations. The exact form of these restrictions is easy to derive; they imply a duality between the (real) 4-form and the (complex) 2-form, where one uses $\mathscr{G}$ instead of ε for dualising,

$$\begin{aligned}*K&=-\tfrac{2c}{3}\mathrm{Re}\,(\mathscr{F}*\bar{\mathscr{G}})\,,\\ *\mathscr{K}&=\tfrac{c}{6}F*\mathscr{G}+\tfrac{ic}{12}(\mathscr{F}\wedge\bar{\mathscr{F}}*\mathscr{G}+\mathscr{F}\wedge\mathscr{F}*\bar{\mathscr{G}})\\ &=\tfrac{c}{6}F*\mathscr{G}-\tfrac{i}{4}\mathscr{F}\wedge*K\end{aligned}\qquad(6.1)$$

(here, $K=\frac{\partial\Phi}{\partial F}$), thus resolving the question of how to obtain the correct number of fields. There are at least three consistency checks involved in the procedure: the first one is the internal consistency of the explicit duality ansatz analogous to (3.5), the second one its integrability, i.e., the possibility of deriving it from an action in the restricted sense described above, and the third that κ-symmetry, which given (6.1) is calculable without knowledge of Φ, requires the same duality.

We have in this paper dealt exclusively with type IIB branes, motivated by the search for SL(2)-invariant formulations. It would be interesting to see if the method applies also in the cases of type IIA and 11 dimensions. This might be done directly in 11 dimensions using the superspace formulation of 11-dimensional supergravity [20, 21] as a background. The membrane [22] only involves a maximum-rank form, and is treated in ref. [19]. It is conceivable that the present method would provide a polynomial κ-symmetric action consistent with a non-linear self-duality for the 11-dimensional 5-brane [23].

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A Some notation

We use the standard conventions for superspace forms with an n-form expanded as

$$\Omega = \frac{1}{n!} dZ^{M_n} \ldots dZ^{M_1} \Omega_{M_1 \ldots M_n} = \frac{1}{n!} E^{A_n} \ldots E^{A_1} \Omega_{A_1 \ldots A_n} \,, \tag{A.1}$$

and the world-volume forms defined analogously. No separate notation is used for pull-backs to the world-volume. The exterior derivative acts from the right as

$$d\Omega = \Omega \wedge \overleftarrow{d} \,, \tag{A.2}$$

and thus obeys

$$d(\Omega_{(m)} \wedge \tilde{\Omega}_{(n)}) = \Omega_{(m)} \wedge d\tilde{\Omega}_{(n)} + (-1)^n d\Omega_{(m)} \wedge \tilde{\Omega}_{(n)} \,. \tag{A.3}$$

The interior product i_V by a supervector field acts similarly. Scalar products of tensors are defined as

$$A \cdot B \equiv \frac{1}{n!} A_{i_1 \ldots i_n} B^{i_1 \ldots i_n} \,, \tag{A.4}$$

and Hodge duality as

$$* \Omega^{i_1 \ldots i_{D-n}} \equiv \frac{1}{n! \sqrt{-g}} \varepsilon^{i_1 \ldots i_n} \Omega_{i_{D-n+1} \ldots i_D} \,, \tag{A.5}$$

with $\varepsilon^{01 \ldots (D-1)} = 1$.

Unlike most authors, we have chosen a convention where complex conjugation does not reverse the order of fermions, for which reason the dimension-0 component of the torsion in (5.4) does not contain a factor i. We find that this convention simplifies reality properties in complex superspace.

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